Water quality failures in distribution networks – risk analysis using fuzzy logic and evidential reasoning

NRCC-50083

Sadiq, R.; Kleiner, Y.; Rajani, B.

A version of this document is published in / Une version de ce document se trouve dans:
Risk Analysis, v. 27, no. 5, Oct. 2007, pp. 1381-1394

Water quality failures in distribution networks - risk analysis using fuzzy logic and evidential reasoning

Rehan Sadiq, Yehuda Kleiner, and Balvant Rajani

Buried Utilities Research
Urban Infrastructure Program
Institute for Research in Construction (IRC)
National Research Council of Canada (NRC), Ottawa, ON, Canada K1A 0R6

Abstract: The evaluation of the risk of water quality failures in a distribution network is a challenging task given that much of the available data are highly uncertain and vague, and many of the mechanisms are not fully understood. Consequently, a systematic approach is required to handle quantitative-qualitative data as well as means to update existing information when new knowledge and data become available.

Five general pathways (mechanisms) through which a water quality failure can occur in the distribution network are identified in this paper. These include contaminant intrusion, leaching and corrosion, biofilm formation and microbial regrowth, permeation, and water treatment breakthrough (including disinfection byproducts formation). The proposed methodology is demonstrated using a simplified example for water quality failures in a distribution network. This paper builds upon the previous developments of aggregative risk analysis approach.

Each basic risk item in a hierarchical framework is expressed by a triangular fuzzy number, which is derived from the composition of the *likelihood* of a failure event and the associated failure *consequence*. An analytic hierarchy process is used to estimate weights required for grouping non-commensurate risk sources. The evidential reasoning is proposed to incorporate newly arrived data for the updating of existing risk estimates. The exponential ordered weighted averaging operators are used for defuzzification to incorporate attitudinal dimension for risk management. It is envisaged that the proposed approach could serve as a basis to benchmark acceptable risks in water distribution networks.

Keywords: Fuzzy logic, evidential reasoning, water quality, distribution networks, exponential ordered weighted average operators, and analytic hierarchy process.

INTRODUCTION

Safety of drinking water is a high priority of water purveyors and stakeholders (owners and customers). A typical modern water supply system comprises the water source (groundwater or surface water including the catchment basin), transmission mains, treatment plants and a distribution network, which includes pipes and distribution tanks. While water quality can be compromised at any component, failure at the distribution level can be extremely critical because it is closest to the point of delivery and, with the exception of a rare filtering device at the consumer level, there are virtually no safety barriers before consumption.

Water quality failures

Water quality is generally defined by a collection of upper and lower limits on selected indicators (contaminants) in the water (Maier, 1999), which can be classified into three broad categories: physical, chemical and biological contaminants. The upper and lower limits are often governed by regulations (Swamee and Tyagi, 2000). A water distribution network acts as a complex reactor in which various processes occur simultaneously. The water quality in the distribution network, which is an outcome of these processes, continuously changes both temporally and spatially. A water quality failure event is often defined as an exceedance of one or more water quality indicators from specific regulations, or in the absence of regulations, exceedance of guidelines or self-imposed, customer-driven limits. Water quality failures in distribution networks can generally be classified into the following major categories or pathways (Kleiner, 1998), also described in Figure 1:

- Contaminant intrusion into the distribution network through system components,
- Biofilm formation and regrowth of microorganisms in a distribution network,
- Water treatment breakthrough of bacteria and/ or chemicals, formation of disinfection byproducts (DBPs),
- Leaching of chemicals, release of corrosion byproducts, and
- Permeation of organic compounds from the soil through system components.

An intrusion of contaminants into the water distribution network can occur through storage tanks (animals, dust-carrying bacteria, infiltration) and pipes. Intrusion through water mains may occur during or after maintenance and repair events, through broken or corroded (pinholes or cracks) pipes and joints/ gaskets, and through cross-connections (Kirmeyer *et al*., 2001). Whenever the water pressure in a pipe is very low or negative, the risk of contamination through backflow or through leaky pipes increases. This can happen when the pipe is de-pressurized for repair or during transient pressures (e.g., when the hydrant is used for fire extinguishing or water hammer events).

Biofilm is a deposit consisting of microorganisms, microbial products and detritus at the surface of pipes or tanks. Biological regrowth may occur when injured bacteria enter from the treatment plant into the distribution network. Under favorable conditions, such as nutrient supply (e.g., organic carbon) in the water and long residence time, these bacteria can attach themselves to surfaces, rejuvenate and grow in storage tanks and on rough inner surfaces of water mains. The regrowth of microorganisms in the distribution network results in an increased chlorine demand, which has two adverse effects: (a) a reduction in the level of free available chlorine may hinder the network's ability to contend with local occurrences of contamination (US EPA, 1999), and (b) an increased level of disinfection to satisfy the chlorine demand of biofilm may result in higher concentrations of disinfection byproducts (DBPs).

Internal corrosion of metallic pipes and plumbing devices may increase the concentration of metal compounds in the water. Different metals go through different corrosion processes, but in general low pH water, high dissolved oxygen, high temperature, and high levels of dissolved solids increase corrosion rates. Metals such as lead and cadmium may leach into the water from pipes, causing significant health effects. Secondary metals such as copper (from home plumbing), iron (distribution pipes) and zinc (galvanized pipes) may leach into water causing taste, odor and color (red or rusty water) problems in addition to some minor health-related risks (Kleiner, 1998). Leaching of chemicals into the water supply can often come from the internal lining and coating of pipes (e.g., volatile organic compounds), causing physico-chemical water quality failure with adverse health and aesthetic consequences.

Permeation is a phenomenon in which contaminants (notably hydrocarbons) from polluted site migrate through the walls of plastic pipes. Three stages are observed in permeation: (a) organic chemicals present in the soil partition between the soil and the plastic wall, (b) the chemicals defuse through the pipe wall, and (c) the chemicals partition between the pipe wall and the water inside the pipe (Kleiner, 1998). In general, the risk of contamination through permeation is relatively small as compared to other mechanisms.

Risk analysis techniques

Commonly, "risk" refers to *the joint probabilities of an occurrence of an event and its consequences* and "risk analysis" refers to a process of *an estimation of the frequency and physical consequences of undesirable events* (Ricci *et al*., 1981). Risk analysis may include a range of techniques from a simple qualitative analysis (e.g., preliminary hazard analysis) to very complex quantitative techniques (e.g., Bayesian networks) for dynamic systems. A brief discussion on some of the risk analysis techniques is provided in this section.

Preliminary hazard analysis (PHA) is a qualitative technique for conducting hazard assessment in chemical process industries. The PHA can identify systems/ processes, which require further examination to control major hazards (Fullwood and Hall, 1988). Hazard and operability study (HAZOP) is a technique also commonly employed in chemical process industries for estimating safety risk and operability improvements (Sutton, 1992). Failure mode and effects analysis (FMEA) is commonly used in reliability engineering to analyze potential failure modes in a system and rank them according to their severity. When the FMEA is extended to criticality analysis, the technique is called failure mode and effects criticality analysis (FMECA) (Chakib *et al*., 1992).

Tree-based (hierarchical) techniques are also widely used to perform risk analysis. A fault tree is a logical diagram, which shows the relation between system failure, i.e. a specific undesirable event in the system, and failures of the components of the system (Vincoli, 1994). Event tree analysis (ETA) is a technique to illustrate the sequence of outcomes, which may arise after the occurrence of a selected initial event (Suokas and Rouhiainen, 1993). Cause-consequence analysis (CCA) combines cause analysis (described by fault trees) and consequence analysis (described by event trees).

Techniques for the analysis of dynamic systems can involve methods such as digraph/ fault graph, dynamic ETA, Bayesian networks, or fuzzy cognitive maps. The digraph/ fault graph technique uses the mathematics and language of graph theory, which constructs the risk model by replacing system elements with AND and OR gates. Bayesian networks (BN) are directed acyclic graphs, in which nodes represent variables and directed arcs describe the conditional dependence relations embedded in the model. Though the conditional probabilities are often difficult to obtain, BNs are considered as one of the most popular dynamic modeling tools (Pearl, 1988). A fuzzy cognitive map (FCM) is an illustrative representation of the complex system uses cause-effect relationships to perform risk analysis (Kosko, 1986). Recently, MacGillivray *et al*. (2006) provided an excellent review of some of these risk analysis and decision making strategies. This review critically analyzes and reports a wide range of research studies, which use above risk analysis techniques primarily focusing on drinking water supply systems.

The quantification of the risk of contamination in water distribution networks is a difficult task. Water distribution networks comprise many (sometimes thousands of) kilometers of pipes of different ages and various materials, which are subjected to varying operational and environmental conditions. In addition, limited performance and deterioration data are available since pipes are buried structures. Finally, some of the failure processes are not well understood and the diagnosis of contamination is very difficult because there is generally a time lag between the occurrence of failure and the time at which the consequences (e.g., outbreaks) are observed.

Both set theory and probability theory are the classical mathematical frameworks for characterizing uncertainties. Since the 1960s, a number of generalizations of these frameworks have been developed to formalize different types of uncertainties. Klir (1999) reported that well-justified measures of uncertainties are available not only in the classical set theory and probability theory, but also in the fuzzy set theory (Zadeh, 1965), possibility theory (Dubois and Parade, 1988), and the Dempster–Shafer (D–S) theory (Dempster, 1968; Shafer, 1976). Klir (1995) proposed a comprehensive general information theory (GIT) to encapsulate these concepts into a single framework and established links among them.

Sadiq *et al*. (2004) developed a hierarchical (or tree-based) structure that broke down the overall risk of water quality failures in a distribution network into basic risk items. Risk was characterized qualitatively (or linguistically) based on fuzzy techniques combined with an analytic hierarchy process (AHP). This paper builds upon the previous developments and addresses four important aspects of the aggregative risk analysis in distribution networks. These aspects are: (a) Risk fuzzification – mapping of triangular fuzzy numbers of basic risk items to 5-tuple fuzzy risk set, (b) Risk aggregation – aggregating fuzzy risk for hierarchical structure (c) Risk updating - using evidential reasoning to fuse newly arrived data (or belief) with existing knowledge, and updating risk estimates at any level in the hierarchical structure, and (d) using exponential ordered weighted average (E–OWA) operators for defuzzification to consider the decision-maker's attitude towards risk (level of optimism) when deriving the final expressions for aggregative risk.

THE PROPOSED FRAMEWORK

In many engineering problems, information about the probabilities of various risk items is vaguely known or assessed. Fuzzy logic provides a language with syntax and semantics to translate qualitative knowledge into numerical reasoning. When conducting risk analysis for complex systems, decision-makers, engineers, managers, regulators and other stakeholders often articulate the risk in terms of linguistic variables like *very high*, *high*, *very low*, *low* etc. The fuzzy-based techniques are able to deal effectively with such vague and imprecise probabilities for approximate reasoning, which subsequently help the decision-making process.

Triangular fuzzy numbers (TFNs) are often used for representing linguistic variables (Lee, 1996). A more comprehensive description of fuzzy-based techniques is not provided in this paper because of space limitations. Interested readers are encouraged to consult excellent texts on this topic written by Klir and Yuan (1995) and Ross (2004).

(a) Risk fuzzification

Let the likelihood (probability) r of failure be defined by the triangular fuzzy number TFN_r and the consequence (or peril) l of failure be defined by TFN_l. Table 1 describes an 11-grade scale for both r and l. Let failure risk be defined by the 5-grade TFN_L, described in Table 2. The definitions of TFNs can be changed or modified based on expert opinion or on Delphi based surveys.

The risk of failure in the probabilistic realm is the joint probability of occurrence and consequences of failure. When the probabilities of occurrence and failure are assumed to be independent of each other, their joint probability is equal to the product of the respective probabilities. Under the same assumption of independence, the fuzzy risk of failure will be calculated as the product of the two fuzzy numbers denoted by r and l. By definition, the product of two TFNs is itself a TFN. Let TFN_r be defined by the members (a_r, b_r, c_r), and TFN_l by (a_l, b_l, c_l). The risk TFN_{rl} for these r and l is calculated by,

$$x = TFN_{rl} = TFN_r \times TFN_l \qquad = (a_r * a_l,\ b_r * b_l,\ c_r * c_l) \tag{1}$$

For example, if an event has a likelihood r as *high* [0.6, 0.7, 0.8] and the peril l is *unimportant* [0.2, 0.3, 0.4], the corresponding risk x will be a TFN_{rl} [0.12, 0.21, 0.32]. There are 5 steps to convert fuzzy number TFN_{rl} into fuzzy risk X - a normalized 5-tuple fuzzy set. These steps are also illustrated in Figure 2.

- Map TFN_{rl} over TFN_L (p = 5-grades defined over the universe of discourse of risk);
- determine the points where TFN_{rl} intersects each TFN_L (Table 2);
- use a *maximum* (*or-type*, *t*-conorm) operator if TFN_{rl} intersects any TFN_L at more than one point (Figure 2);
- establish a set of intersecting points (or the maximum thereof if more than one) that defines a non-normalized 5-tuple fuzzy set, X_L, (e.g. in Figure 2, X_L is [0.38, 0.88, 0.2, 0, 0], which are the memberships of X_L to the grades *very low*, *low*, *medium*, *high* and *very high* risk, respectively); and
- normalize X_L to obtain fuzzy set X, where membership μ_p of X_L is transformed to μ_p^N of X by dividing each μ_p by the cardinality C (sum of all memberships in a fuzzy set).

$$\mu_p^N = \frac{\mu_p}{\sum_{p=1}^{n} \mu_p} = \frac{\mu_p}{C} \tag{2}$$

In the example of Figure 2, the fuzzy set X is [0.26, 0. 6, 0.14, 0, 0], and can also be expressed as, $X = \left[\frac{0.26}{VL}, \frac{0.6}{L}, \frac{0.14}{M}, \frac{0}{H}, \frac{0}{VH}\right]$.

(b) Risk aggregation

Figure 3 illustrates the basic building blocks of the proposed hierarchical structural model for the risk aggregation. Each risk item is partitioned into its contributory factors, which are also risk items, and each of those can be further partitioned into lower level contributory factors. A unit that consists of a risk factor (“parent”) and its contributory factors (“children”) is called a “family”.

A risk unit with no children is called “basic risk item”, while the term *risk item* is used for all elements with offspring. The notation used for a risk item is $X_{i,j}^{k}$, where *i* is the ordinal number of risk item *X* in the current generation; *j* is the ordinal number of the parent (in the previous generation); and *k* is the generation order of *X*. The indices *i, j, k* are used for risk item attributes as well, e.g., in the table of Figure 3, the factors $r_{i,j}^{k}$ and $l_{i,j}^{k}$ denote *likelihood* and *peril* (respectively) for the risk item $X_{i,j}^{k}$.

Various “inferencing” methods can be used to aggregate fuzzy sets, however in this study, “inferencing” through *weighted average* is proposed to determine the aggregative risk. The *weighted average* inferencing refers to *sum-prod* fuzzy compositional operator. There are different types of fuzzy composition operators available like *max-min* (reflects low uncertainty range), *sum-prod* (reflects high uncertainty range) and *mix* of both *max-min* and *sum-prod*. These compositional operators express the various degrees of *and-ness* and *or-ness* in the application of fuzzy sets. Logical operators like *max-min* are more restrictive than, say, *sum-prod* and *max-prod.* For simplicity the *weighted average* (*sum-prod*) is used in this study (Sadiq *et al*., 2003).

A weighting scheme is required when the respective contributions of sibling risk items towards their parent have non-commensurate units. Figure 3 shows a general case where weights are assigned to each risk item. The notation used is $w_{i,j}^{k}$, which denotes the weight of $X_{i,j}^{k}$ relative to its siblings. When the respective contributions of sibling risk items towards their parent have commensurate units, then all the siblings have equal weights, $w_{i,j}^{k}$, which means that they can be ignored altogether. Saaty (2001) described in detail the analytic hierarchy process to derive weights. These weights are normalized to a sum of unity, such that in any generation (*k*), for *n* siblings with parent *j*, a set of weights can be written as,

$$w_{i,j}^{k} = \left[w_{1,j}^{k}, w_{2,j}^{k}, ..., w_{n,j}^{k}\right] \quad \text{where} \qquad \sum_{i=1}^{n} w_{i,j}^{k} = 1 \qquad (3)$$

The process of evaluating aggregative risk in a “family” with an aggregative structure is described using the family (Figure 3) of $X_{2,1}^{2}$ (parent) and $X_{3,2}^{3}$, $X_{4,2}^{3}$, $X_{5,2}^{3}$ (children) as an example. For each of the sibling risk items, the likelihood *r* and peril *l* are assigned from the 11-grade scaling system (Table 1). $TFN_{rl}(x)$ is the product of two fuzzy numbers TFN_r and TFN_l (Equation 1), which is then mapped over TFN_L to obtain the 5-tuple fuzzy set X_L (a non-normalized fuzzy set for risk). X_L is then normalized to obtain the 5-tuple fuzzy sets $X_{3,2}^{3}$, $X_{4,2}^{3}$, $X_{5,2}^{3}$, representing the risk contribution of each of the siblings towards their parent. For ease of manipulation these 5-tuple sets can be arranged in a fuzzy assessment matrix, which is a 3 × 5 matrix $F(X_{i,2}^{3})$. The AHP is then applied, weights $w_{3,2}^{3}$, $w_{4,2}^{3}$, and $w_{5,2}^{3}$ are evaluated and arranged into a 3-member vector. The aggregative risk (or parent) of the three siblings is the cross product of weights vector and the assessment matrix, yielding a 5-tuple fuzzy set $X_{2,1}^{2}$,

$$X_{2,1}{}^{2} = \left[w_{3,2}{}^{3}, w_{4,2}{}^{3}, w_{5,2}{}^{3}\right] \times F(X_{i,2}{}^{3}) = \left[\mu_1^N, \mu_2^N, \ldots, \mu_5^N\right] \tag{4}$$

where μ_p^N (p = 1, 2,…, 5) are the membership values of the aggregated risk with respect to the 5-grade risk scale.

It should be noted that the process of evaluating r and l and mapping the product risk onto the 5-grade risk scale is necessary only for basic risk items, i.e. those risk items, which do not have children. All subsequent risk aggregations from one generation to the next are determined by only applying equation (4) using the appropriate relative weights. Consequently it is useful to use notation that distinguishes between basic and non-basic risk items. In the remainder of this paper, the notation for a basic risk item will include an apostrophe at the generation index, i.e., if item $X_{4,2}^{3}$ is a basic risk item, it will be denoted by $X_{4,2}^{3'}$.

(c) Risk updating using evidential reasoning (D–S rule of combination)

In classical Bayesian inference, the sum of probabilities of any set A and its complement, $p(A) + p(\neg A) = 1$. This implies that knowledge about A can be used to derive a belief about its complement. For example, let $\Theta = \{A, B, C\}$, be a *frame of discernment* (also called a *universe of discourse* meaning all possible outcomes), and let the evidence $p(A) = a$. According to equal noninformative priors (*Laplace Principle of Insufficient Reason*), then $p(B) = p(C) = 0.5(1 - a)$, i.e., the probability of the complement of A will be equally distributed in subsets B or C.

In contrast to the above, Dempster–Shafer (D–S) theory is based on the premise that missing evidence (or lack of knowledge, or ignorance) about $\neg A$ does not justify an assumption about probabilities of B and C (Alim, 1988). The D–S theory can be interpreted as a generalization of probability theory, where probabilities are assigned to *subsets* as opposed to mutually exclusive *singletons*. For example, let the universal set $\{L, M, H\}$ contain three basic elements. The *frame of discernment* Θ comprise all combinations of the basic elements in the universal set, in our example, the 8 subsets of Θ are ϕ, $\{L\}$, $\{M\}$, $\{H\}$, $\{L, M\}$, $\{L, H\}$, $\{M, H\}$, and $\{L, M, H\}$. The subset $\{L, M\}$ means $\{L\}$ or $\{M\}$. Consequently, the subset $\{L, M, H\}$ represents a complete ignorant situation (i.e., we do not know which will be the outcome, it can be any of *L*, *M* or *H*). It can be shown that Θ comprises 2^n subsets where *n* is number of basic elements.

The D–S theory defines a *basic probability assignment* (*bpa* is denoted by *m*). Let evidence *A* be a subset of Θ. The *bpa* $m(A)$ is defined over the interval [0, 1]. The *bpa* of a null set $m(\phi) = 0$. The complement of *A* is always attributed to the complete ignorance, i.e., subset Θ. For example, let evidence $A = \{\{\mathrm{L}\}, \{L, M\}\}$ so that $m(A)_{\mathrm{L}} = 0.6$ and $m(A)_{\mathrm{L,M}} = 0.2$, then $m(A) = 0.8$ and $m(A)_{\Theta} = 1 - 0.8$. For a given *basic probability assignment m*, every non-ignorant subset *A* (i.e., $m(A) \neq 0$) is called *focal element*, e.g., in the example above, $m(A)_{\mathrm{L}}$ and $m(A)_{\mathrm{L,M}}$ are focal elements.

The D–S rule of combination defines how to combine evidence obtained from two or more sources. It strictly emphasizes agreements between multiple sources and ignores all conflicting evidence through *normalization*. A strict *conjunctive* operation (and-type or intersection type operator) using a “product” is used to combine the evidences. For example, if *B* and *C* are two sources of information, the D–S rule of combination establishes the joint *bpa* $m_{1\text{-}2}(A)$ from the aggregation of *bpas* $m_1(B)$ and $m_2(C)$,

$$m_{1-2}(A) = \frac{\sum_{B\cap C=A} m_1(B) m_2(C)}{1-K} \quad when \quad A \neq \phi; \qquad m_{1\text{-}2}(\phi) = 0; \tag{5}$$

where $K = \sum_{B\cap C=\phi} m_1(B)\, m_2(C)$

K is the *degree of conflict* between two bodies of evidence. It can be shown that the denominator (1-*K*) in equation (5) is a normalization factor, which always brings the sum of all $m_{1\text{-}2}(A)$ values to unity. The above equations can be rewritten as,

$$m_{1-2}(A) = \frac{\sum_{B\cap C=A} m_1(B)\, m_2(C)}{\sum_{B\cap C\neq\phi} m_1(B)\, m_2(C)} \tag{6}$$

Zadeh (1984) identified a serious shortcoming in the D–S rule of combination due to the use of strict conjunctive operator (*product*). Sentz and Ferson (2002) have provided an excellent review of various techniques to overcome this discrepancy. Recently, Yager (2004) proposed the use of disjunctive operators (or-type operator, denoted by ⊕), according to which, equation (6) can be modified as,

$$m_{1-2}(A) = \frac{\sum_{B\cap C=A} \oplus [m_1(B), m_2(C)]}{\sum_{B\cap C\neq\phi} \oplus [m_1(B), m_2(C)]} \tag{7}$$

A disjunctive logic using "*max*" operator can be used in equation (7). The approach described above implicitly assumes that all sources of information are equally credible. Yager (2004) suggested a *credibility transformation function*, which discounts evidence with a credibility factor (α) and distributes the remaining evidence (1-α) equally among *n* elements.

$$m(A)_{\alpha} = m(A)\cdot\alpha + \frac{1-\alpha}{n} \tag{8}$$

For example, assume that the evidence obtained from two different sources for risk item $X_{1,1}^2$ are represented by fuzzy sets $m_1(X_{1,1}^2)$ = [0.5, 0.5, 0, 0, 0] and $m_2(X_{1,1}^2)$ = [0, 0.6, 0.4, 0, 0]. Assume further that the corresponding credibility factors are $\alpha_1 = 1$ and $\alpha_2 = 0.5$, respectively. The bodies of evidence are adjusted and the D–S rule of combination is used to obtain the fuzzy set $m_{1\text{-}2}(X_{1,1}^2)$ = [0.33, 0.33, 0.2, 0.07, 0.07]. In the hierarchical structure described earlier, D–S updating can be done at any level of the hierarchy when new evidence is available, however, it is expected that D–S updating will be done mainly at the level of basic risk items. This process is thus used to combine the new information with prior information.

(d) Risk management (using defuzzification)

In the first generation of the aggregative structure (i.e., the head of the pyramid), the final aggregative risk is a fuzzy set that can be defuzzified to provide a single (crisp) measure of the risk, using one of the several defuzzification techniques described in Chen and Hwang (1992). Lee (1996) proposed a simple defuzzification technique as follows,

$$\text{Defuzzified risk} = L_P \cdot X_{1,0}^{1} \tag{9}$$

which means that the defuzzified risk is calculated as a dot product of vector L_P and the fuzzy number $X_{1,0}^1$, where L_P (given in Table 2) is the 5-tuple vector representing centroid values of *p* linguistic risk constants. The crisp value of risk can be misleading because of associated uncertainties and subjectivity in the use of any defuzzification technique. An attitudinal dimension can be introduced in the defuzzification of risk to alleviate (or at least have control over) this issue.

In the proposed framework the ordered weighted operators (OWA) as described by Yager (1988) is selected as the method to consider the attitude of the decision-maker in defuzzifying the final risk value. The OWA method was used for applications in decision-making (Engemann *et al*., 1996), expert systems (Kacprzyk, 1990), and fuzzy systems (Yager and Filev, 1994). A number of approaches have been suggested to obtain OWA weights (Yager, 1993). O' Hagan (1988), for example, calculated the vector of the OWA weights for a predefined *orness* (level of optimism) by maximizing the entropy of the OWA weights using linear programming. Another (simpler) way of obtaining OWA weights involves the using of exponential OWA (E–OWA) operators, which represent a simple relationship between the *orness* and a parameter β (Filev and Yager, 1998). The E–OWA weights are defined as follows:

$$w_1 = \beta;\ w_2 = \beta(1 - \beta);\ \ldots \qquad w_{n-1} = \beta(1 - \beta)^{n-2} \ \text{and} \qquad w_n = (1 - \beta)^{n-1}\ ;\ 0 \le \beta \le 1 \quad (10)$$

where n is the granularity of fuzzy risk. Once the weights w_p (p = 1, 2, …, n) are determined, the crisp value of the fuzzy risk can be calculated by

$$\text{Defuzzified risk} = \sum_{p=1}^{n} w_p \mu_p^N \quad (11)$$

where μ_p^N are the normalized membership values of the fuzzy risk to the risk levels, which are arranged in a decreasing order of importance (i.e., VH, H, M, L, VL). Parameter β is determined based on the decision-maker's chosen optimism level or *orness*. The *orness* of an E-OWA operator takes on a value between zero (pessimistic) and unity (optimistic) and is related to parameter β as follows

$$Orness = \frac{1}{n-1} \sum_{p=1}^{n} (n-p) w_p \quad (12)$$

Figure 4 illustrates some characteristic curves of *orness* versus β for selected levels of granularity, as calculated using equations (10) and (12). In summary, the steps to estimate risk for a given level of optimism are:

- Assume a level of optimism (*orness*);

- calculate β using equations (10) and (12) or read the value of β from the characteristic curve (Figure 4) with the appropriate number of granulars;
- determine weights using equation (10); and
- determine crisp risk estimates using equation (11).

The risk estimates of alternative strategies at a desired level of optimism can be associated to cost-benefit analysis. The more optimistic the attitude, the higher the willingness to take risks and the lower the cost of risk mitigation. Conversely, lower optimism results in a conservative approach, which involves higher costs. Figure 5 provides a block diagram that illustrates the proposed framework.

WATER QUALITY FAILURE IN DISTRIBUTION NETWORKS

Figure 6 shows a simplified hierarchical structure for water quality failure. A detailed discussion on each type of water quality failure can be found in Sadiq *et al*. (2004). This structure is used to demonstrate the aggregative risk framework introduced in the previous section. Table 3 lists 17 basic risk items for the proposed structure. These basic risk items are positioned in the fourth and the third generations of the hierarchical structure, and are grouped into the third and the second generations (respectively) risk attributes, which in turn are grouped further up the hierarchy. The weight matrices $w_{i,j}^{k}$ for each set of siblings were developed using the AHP technique as discussed earlier.

The process of basic risk evaluation and subsequent risk aggregation through all the generations were performed as described in the previous section. The final aggregated risk (first generation) was obtained $X_{1,0}^{\;\;1} = \left\{ \frac{0.38}{VL}, \frac{0.43}{L}, \frac{0.19}{M}, \frac{0.01}{H}, \frac{0}{VH} \right\}$ and is plotted in Figure 7. The final defuzzified aggregative risk was determined for two levels of *orness* (optimism) 0.6 and 0.8 (Figure 4). The crisp (defuzzified) risk estimates varied between 0.14 and 0.06 for low and high optimistic attitudes, respectively.

In the context discussed here, the D-S updating is demonstrated by reassessing risk based on available new evidence on microbial contamination. This evidence could consist of information such as an increase in off-the-shelf sales of gastrointestinal medication or additional cases reported at local clinics/pharmacies. The new evidence $m_2(X_{6,3}^{3'})$ was expressed as [0, 0, 0.2, 0.8, 0], with a credibility (α_2) of 1. The old evidence $m_1(X_{6,3}^{3'})$ was a 5-tuple fuzzy set [0.58, 0.42, 0, 0, 0] which was assigned a credibility (α_1) of 0.8 with respect to the new evidence. The D–S rule of combination was used to update this risk item and the value of $m_{1\text{-}2}(X_{6,3}^{3'})$ changed to [0.26, 0.21, 0.05, 0.1, 0.37]. The aggregative risk analysis was repeated to obtain a final aggregative risk $X_{1,0}^{1}$ = [0.33, 0.4, 0.19, 0.02, 0.05]. After defuzzification, the crisp risk estimate changed from 0.14 to 0.15 for the low optimistic attitude level and from 0.06 to 0.09 for the high optimistic attitude level. The results of pre- and post-update possibility mass functions are compared in Figure 7.

SUMMARY AND CONCLUSIONS

Water quality in the distribution network is a complex issue, for which available data are scarce and often highly uncertain, imprecise and vague. In addition, there is a high spatial and temporal variability in water quality may occur, and many of the controlling processes are not currently well understood. A comprehensive framework of risk analysis was proposed for water quality failures in the distribution network. The advantages of the framework are:

- It enables the synthesis of both quantitative and qualitative information into a single framework;
- it can explicitly consider and propagate uncertainties, for which probability distributions are not known;
- it is modular and scalable; and new knowledge and information can be accommodated at any stage and in any form. For example, vulnerability to terrorist acts (safety related risk), hydraulic failure, financial risk etc. can be part of this framework;
- it has ability update information based on newly arrived evidence;
- more data results in less uncertainty, which when propagated through the hierarchical structure, can result in reduced aggregative risk. The proposed approach can help pinpoint (identify) those areas where more data would yield the highest benefits;
- it can be used for cost-benefit analysis to facilitate efficient budget allocation and prioritize attention to those areas which have the most adverse impact on total water distribution network risk; and

- it is easily programmable for a computer application and can become a risk analysis tool for a water distribution network.

The limitations of the proposed method are:

- It may be sensitive to the selection of aggregation operators. Different mathematical operators can be used for different segments of the model and trial and error approach can be used to avoid exaggeration and/or eclipsing. Exaggeration occurs when all basic risk items are of relatively low risk, yet the final aggregative risk comes out unacceptably high. Eclipsing occurs when one or more of the basic risk items are of relatively high risk, yet the estimated aggregative risk comes out as unacceptably low.
- This framework supports both qualitative and quantitative data. Some data may be supported by rigorous observations, while other data may be based on beliefs that are loosely supported by anecdotal-information. These two types of data should have different weights in the aggregation process. The hierarchical structure in its current form does not address this need to distinguish between data obtained from sources with different reliabilities.

The structure presented in this paper is but a simplified demonstration of the approach. A comprehensive structure would require a major effort, including the collaboration of several experts with knowledge in several disciplines.

In the model development stages, the final aggregative risk value is expected to have limited meaning for the acceptability level of the risk to the general public. It is envisaged that as the proposed hierarchical structure is developed, risk items are populated and subsequently improved upon (using newly obtained data/evidence), the developers and the guardians of the water distribution networks will gain insight into acceptable risk levels as they are manifested in the final fuzzy and/ or defuzzified risk values. In the longer term, this approach could serve as a basis to benchmark acceptable risks in water distribution networks. A collaborative research project titled “Effect of aging water mains on water quality in the distribution systems” by American Water Works Association Research Foundation (AwwaRF) and National Research Council Canada (NRC) is dealing with this issue. The result of this research project will be disseminated in coming years.

REFERENCES

Alim, S. 1988. Application of Dempster–Shafer theory for interpretation of seismic parameters, *ASCE Journal of Structural Engineering*, 114(9): 2070-2084.

Chakib, K-Z., Alfred Z.K. and Fleming, P.V. 1992. A smart failure mode and effect analysis package, *Proceedings Annual Reliability and Maintainability Symposium*, pp. 414-421.

Chen, S.J., and Hwang, C.L. 1992. *Fuzzy multiple attribute decision-making*, Springer-Verlag, NY.

Dempster, A. 1968. A generalization of Bayesian inference, *Journal of Royal Statistical Society*, Series B 30: 205-247.

Dubois, F., and Parade, H. 1988. *Possibility theory*: *an approach to computerized processing of uncertainty*, Plenum Press, NY.

Engemann, K.J., Miller, H.E., and Yager, R.R. 1996. Decision making with belief structure: an application in risk management, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 4: 1-6.

Filev, D.P. and Yager, R.R. 1998. On the issue of obtaining OWA operator weights, *Fuzzy Sets and Systems*, 94: 157-169.

Fullwood, R.R., and Hall, R.E. 1988. *Probabilistic risk assessment in the nuclear power industry*, 1st Ed. Pergamon Press.

Kacprzy, J. 1990. Inductive learning from considerably erroneous examples with a specificity based stopping rule, *Proceedings of International Conference on Fuzzy Logic and Neural Networks*, Izuka, Japan, p. 819.

Kirmeyer, G.J., Friedman, M., Martel, K., and Howie, D. 2001. *Pathogen intrusion into distribution system*, AwwaRF, Denver, CO, USA.

Kleiner, Y. 1998. Risk factors in water distribution systems, *British Columbia Water and Waste Association 26th Annual Conference*, Whistler, B.C., Canada.

Klir, G.J., and Yuan, B. 1995. *Fuzzy sets and fuzzy logic - theory and applications*, Prentice- Hall, Inc., Englewood Cliffs, NJ, USA.

Klir, J.G. 1995. Principles of uncertainty: what are they? why do we need them?, *Fuzzy Sets and Systems*, 74: 15-31.

Klir, J.G. 1999. On fuzzy set interpretation of possibility theory, *Fuzzy Sets and Systems*, 108: 263-273.

Kosko, B. 1986. Fuzzy cognitive maps, *International Journal of Man-Machine Studies*, 24: 65-75.

Lee, H.-M. 1996. Applying fuzzy set theory to evaluate the rate of aggregative risk in software development, *Fuzzy Sets and Systems*, 79: 323-336.

MacGillivray, B.H., Hamilton, P.D., Strutt, J.E., and Pollard, S.J.T. 2006. Risk analysis strategies in the water utility sector: an inventory of applications for better and more credible decision making, *Critical Reviews in Environmental Science and Technology*, 36(2): 85-139.

Maier, S.H. 1999. *Modeling water quality for water distribution systems*, Ph.D. thesis, Brunel University, Uxbridge.

O' Hagan, M. 1988. Aggregating template rule antecedents in real-time expert systems with fuzzy set logic, *Proceedings 22nd Annual IEEE Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, pp. 681-689.

Pearl, J. 1988. *Probabilistic reasoning in expert systems*, San Mateo, CA: Morgan Kaufmann.

Ricci, P.F., Sagen, L.A., and Whipple, C.G. 1981. *Technological risk assessment series* E: Applied Series No.81, NATO Asi Series, Erice (Italy), ISBN 90-247-2961-0.

Ross, T. 2004. *Fuzzy logic with engineering applications*, 2nd Edition, John Wiley & Sons, NY.

Saaty, T.L. 2001. How to make a decision? Chapter 1, *Models, methods, concepts and applications of the analytic hierarchy process*, (Ed.) Saaty, T.L., and Vargas, L.G., Kluwer International Series.

Sadiq, R., Kleiner, Y., and Rajani, B.B. 2003. Forensics of water quality failure in distribution system - a conceptual framework, *Journal of Indian Water Works Association*, 35(4): 267-278.

Sadiq, R., Kleiner, Y., and Rajani, B.B. 2004. Aggregative risk analysis for water quality failure in distribution networks, *AQUA - Journal of Water Supply: Research & Technology*, 53(4): 241-261.

Sentz, K. and Ferson, S. 2002. Combination of evidence in Dempster-Shafer theory, SAND 2002-0835.

Shafer, G. 1976. *A mathematical theory of evidence*, Princeton University Press, Princeton, N.J.

Suokas, J. and Rouhiainen, V. 1993. *Quality management of safety and risk analysis*, Elsevier Science Publishers.

Sutton, I.S. 1992. *Process reliability and risk management*, 1st Ed. Van Nostrand Reinhold, 1992.

Swamee, P.K., and Tyagi, A. 2000. Describing water quality with aggregate index, *ASCE Journal of Environmental Engineering*, 126(5): 451-455.

US EPA 1999. *Microbial and disinfection byproduct rules – simultaneous compliance guidance manual*, United States Environmental Protection Agency, EPA 815-R-99-015.

Vincoli, J.W. 1994. *Basic guide to accident investigation and loss control*, Van Nostrand Reinhold.

Yager, R.R. 1988. On ordered weighted averagng aggregation operators in multi-criteria decision making, *IEEE Transactions on Systems, Man and Cybernatics*, 18: 183-190.

Yager, R.R. 1993. Families of OWA operators, *Fuzzy Sets and Systems*, 59: 125-148.

Yager, R.R. 2004. On the determination of strength of belief for decision support under uncertainty – Part II: fusing strengths of belief, *Fuzzy Sets and Systems*, 142: 129-142.

Yager, R.R. and Filev, D.P. 1994. Parameterized "andlike" and "orlike" OWA operators, *International Journal of General Systems*, 22: 297-316.

Zadeh, L.A. 1965. Fuzzy sets, *Information and Control*, 8: 338-353.

Zadeh, L.A. 1984. Review of books: A mathematical theory of evidence, *The AI Magazine*, 5(3): 81-83.

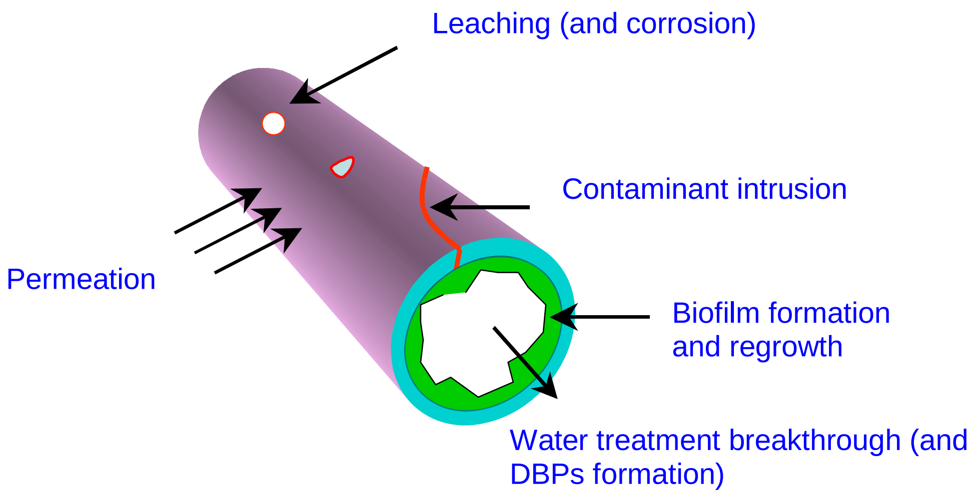
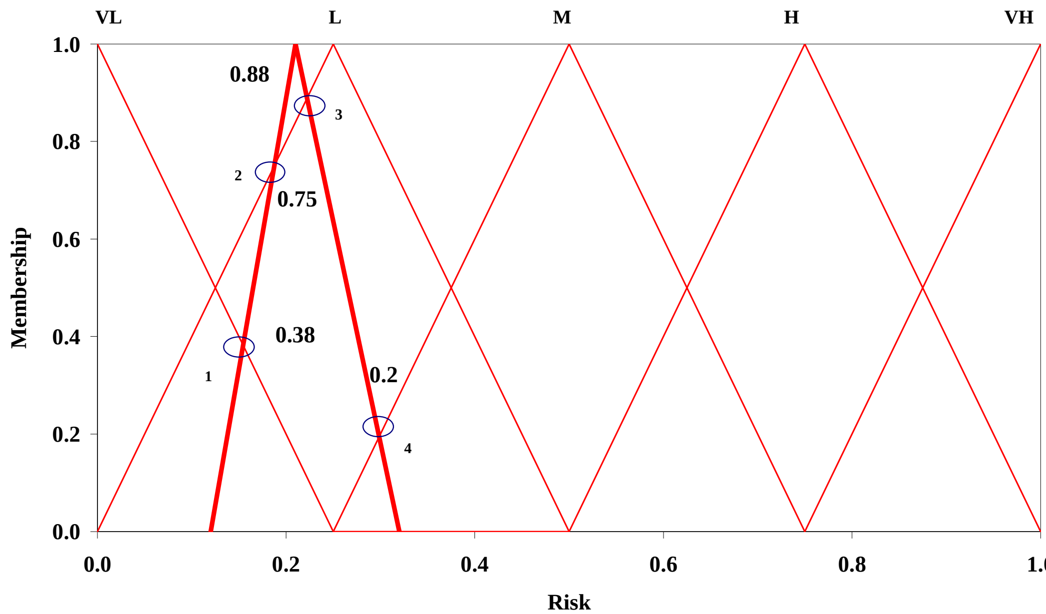
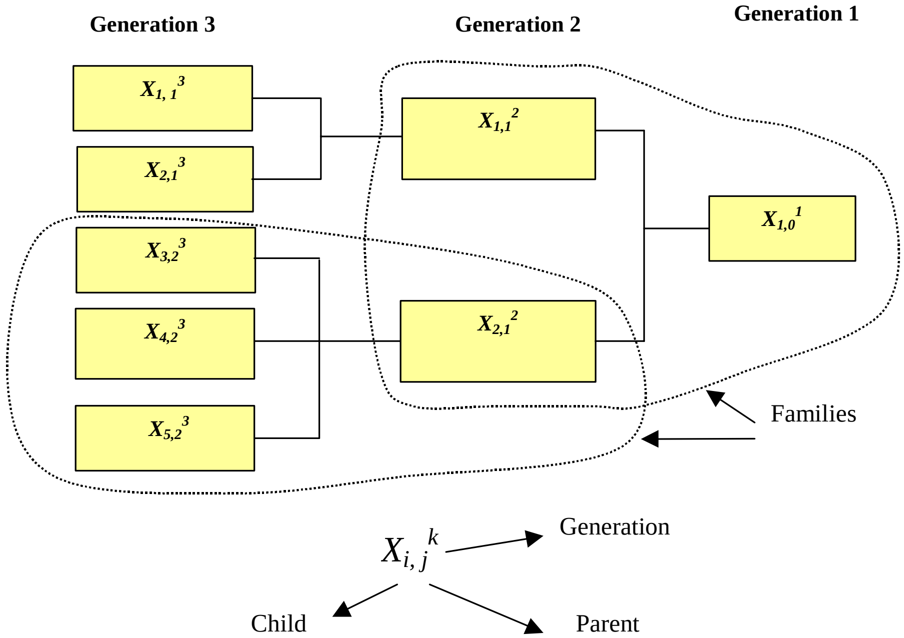


Figure 1. Pathways for water quality failures in water distribution networks



$x = TFN_{rl}$ =	[0.12, 0.21, 0.32]				
p	VL	L	M	H	VH
TFN_L	[0, 0, 0.25]	[0, 0.25, 0.5]	[0.25, 0.5, 0.75]	[0.5, 0.75, 1]	[0.75, 1, 1]
Inference	0.38	*max*(0.75, 0.88)	0.2	0	0
X_L =	[0.38, 0.88, 0.2, 0, 0] (Cardinality, C = 1.46) 5-tuple fuzzy set represents memberships μ_p to qualitative risk levels VL, L, M, H and VH				
X =	[0.26, 0.60, 0.14, 0, 0]				

Figure 2. Estimating 5-tuple fuzzy set of risk



$r_{i,j}^k$	$l_{i,j}^k$	*$TFN_{rl_{i,j}}^3$	** $X_{i,j}^3$	$w_{i,j}^3$	$X_{i,j}^2$	$w_{i,j}^2$	† $X_{i,j}^1$
$r_{1,1}^3$	$l_{1,1}^3$	$x_{1,1}^3$	$X_{1,1}^3$	$w_{1,1}^3$	$X_{1,1}^2$	$w_{1,1}^2$	$X_{1,0}^1$
$r_{2,1}^3$	$l_{2,1}^3$	$x_{2,1}^3$	$X_{2,1}^3$	$w_{2,1}^3$			
$r_{3,2}^3$	$l_{3,2}^3$	$x_{3,2}^3$	$X_{3,2}^3$	$w_{3,2}^3$	$X_{2,1}^2$	$w_{2,1}^2$	
$r_{4,2}^3$	$l_{4,2}^3$	$x_{4,2}^3$	$X_{4,2}^3$	$w_{4,2}^3$			
$r_{5,2}^3$	$l_{5,2}^3$	$x_{5,2}^3$	$X_{5,2}^3$	$w_{5,2}^3$			

*the risk TFN_L; ** normalized 5-tuple fuzzy set for risk; † for parent of generation 1, and $j = 0$

Figure 3. A hierarchical structure for the estimation of aggregative risk

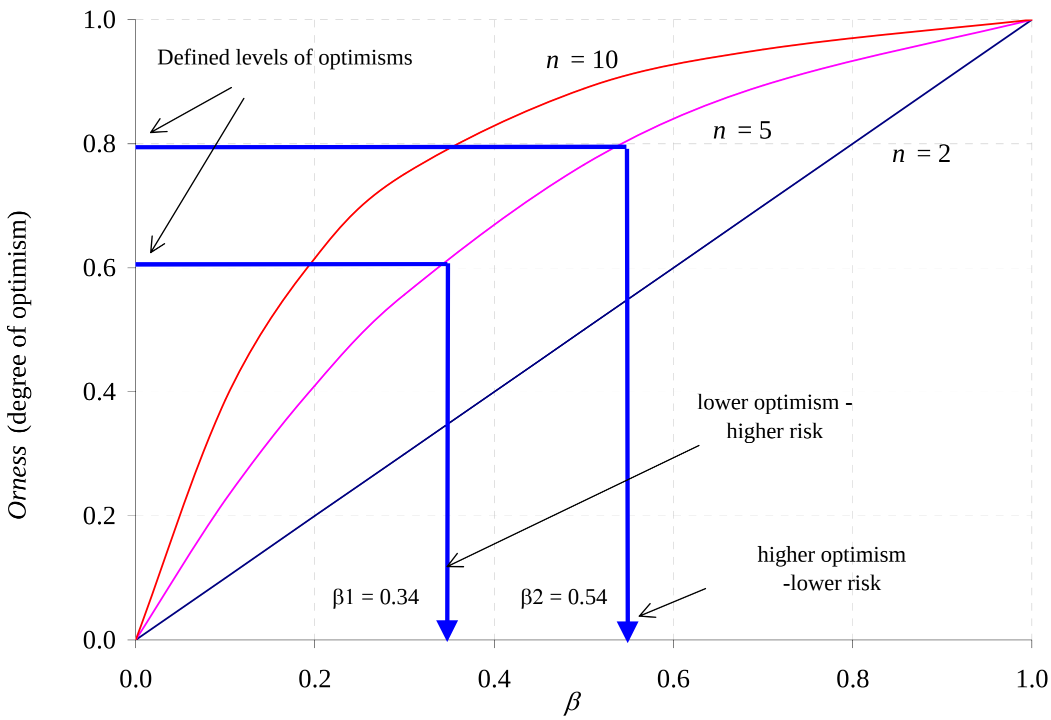


Figure 4. Characteristic curves for representing functional relationships between *orness* and parameter β to determine E-OWA and crisp risk estimates

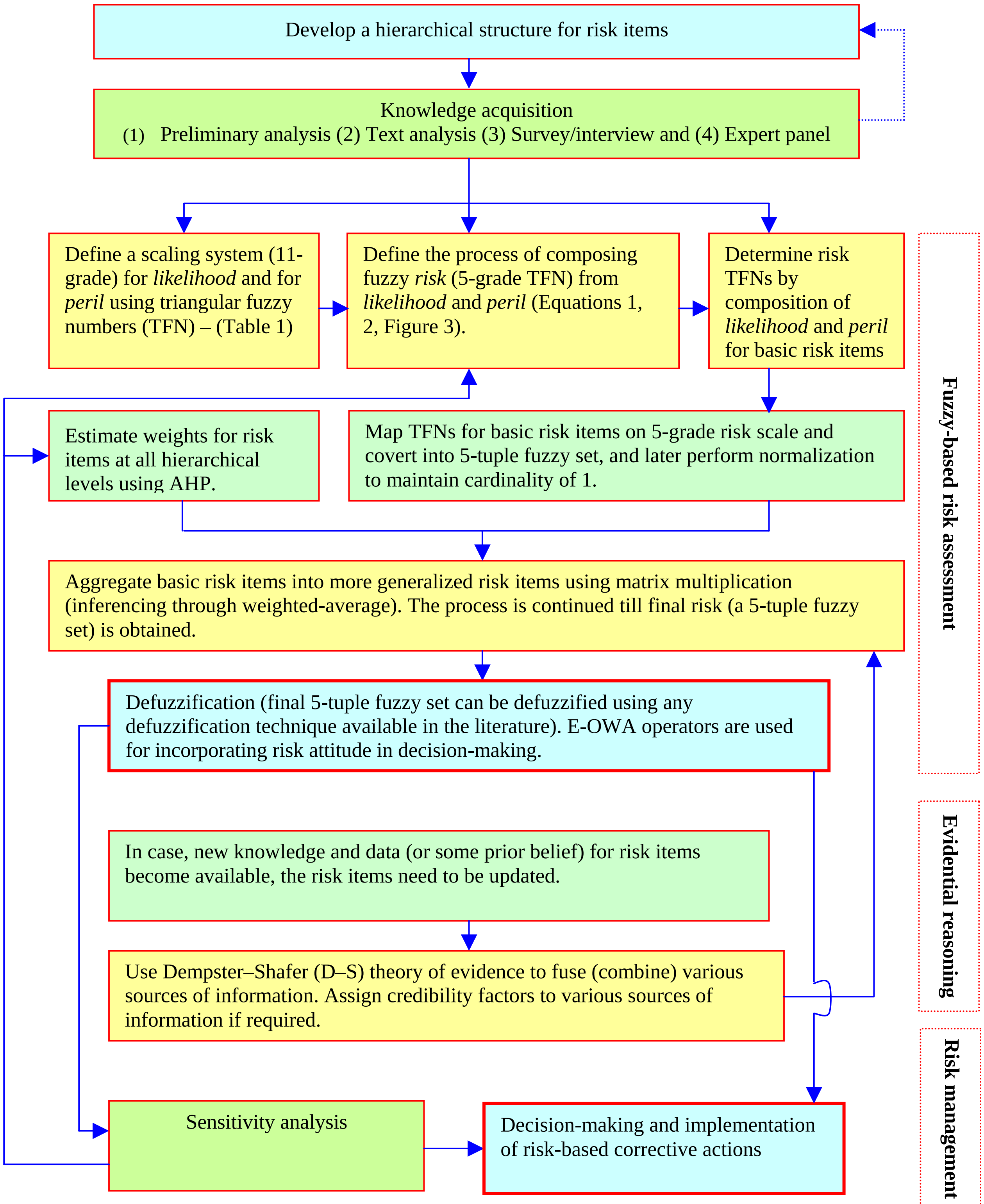


Figure 5. Proposed framework for aggregative risk analysis

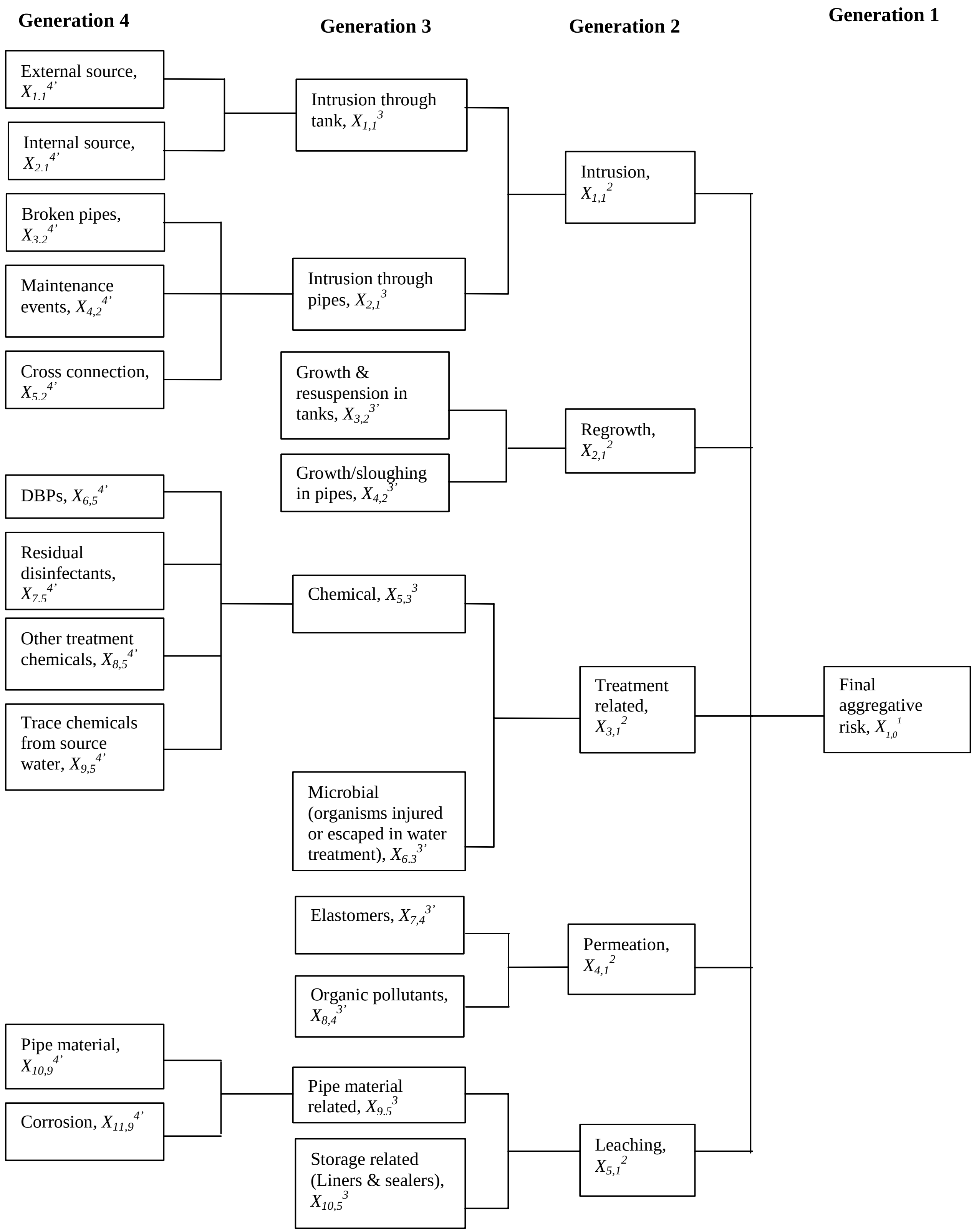


Figure 6. Hierarchical structure for aggregative risk of water quality failure (Sadiq *et al*., 2004)

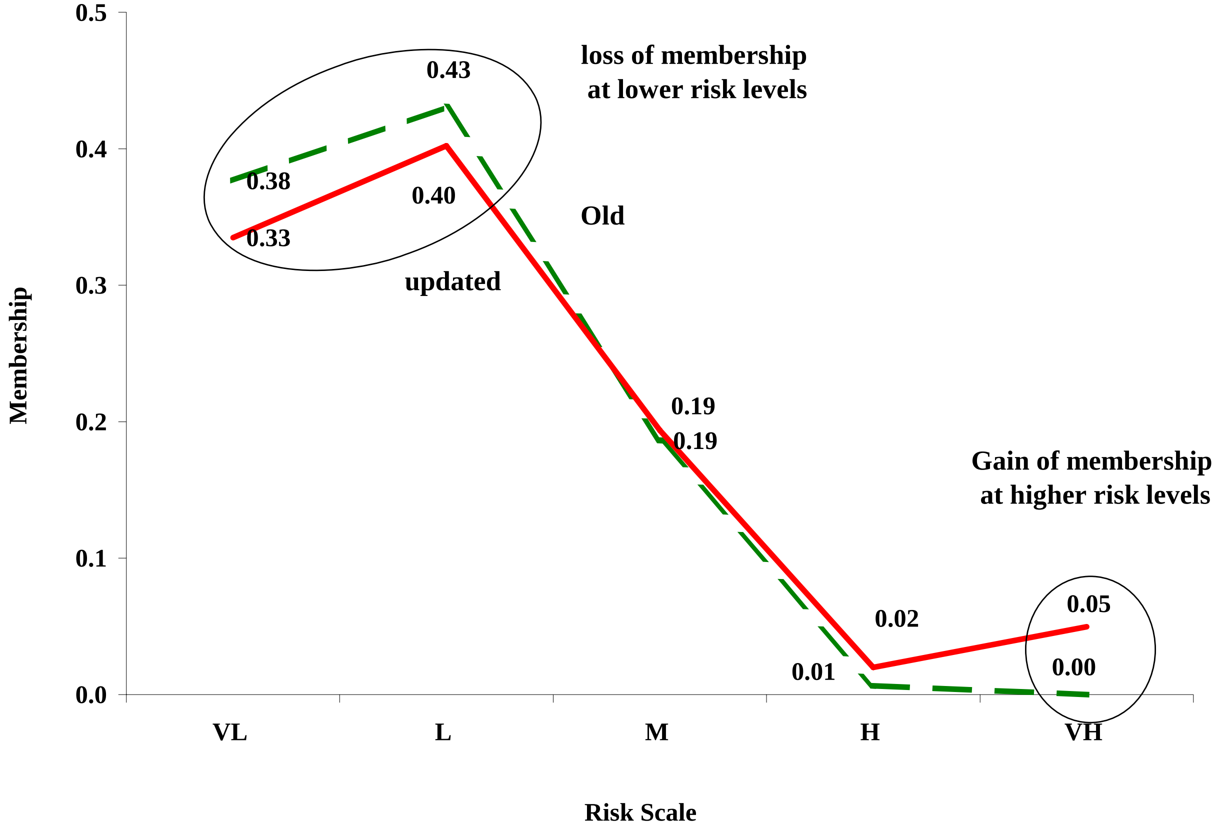


Figure 7. Possibility mass functions for final aggregative risk of water quality failure

Table 1. Linguistic definitions of grades (granulars) using TFNs for *likelihood* and *peril**

Granular (q)	Qualitative scale for likelihood of risk (r)	Qualitative scale for peril of risk (l)	Triangular fuzzy number (TFN_r or TFN_l)
1	Absolutely low	Absolutely unimportant	[0, 0, 0.1]
2	Extremely low	Extremely unimportant	[0, 0.1, 0.2]
3	Quite low	Quite unimportant	[0.1, 0.2, 0.3]
4	Low	Unimportant	[0.2, 0.3, 0.4]
5	Mildly low	Mildly unimportant	[0.3, 0.4, 0.5]
6	Medium	Neutral	[0.4, 0.5, 0.6]
7	Mildly high	Mildly important	[0.5, 0.6, 0.7]
8	High	Important	[0.6, 0.7, 0.8]
9	Quite high	Quite important	[0.7, 0.8, 0.9]
10	Extremely high	Extremely important	[0.8, 0.9, 1]
11	Absolutely high	Absolutely important	[0.9, 1, 1]

* For absolute zero and one, "none" and "certain" qualitative scale can be used, respectively. The TFNs for these qualitative scale are (0, 0, 0) and (1, 1, 1), respectively for both *likelihood* and *peril*.

Table 2. Linguistic definitions of grades (granulars) using TFNs for risk

Granulars (p)	Qualitative scale for risk level (L)*	Triangular fuzzy number (TFN_L)	Centroid (L_P)
1	Very low	[0, 0, 0.25]	0.08
2	Low	[0, 0.25, 0.5]	0.25
3	Medium	[0.25, 0.5, 0.75]	0.5
4	High	[0.5, 0.75, 1]	0.75
5	Very high	[0.75, 1, 1]	0.92

* For absolute zero and one, "none" and "certain" qualitative scale can be used, respectively. The TFNs for these qualitative scale are (0, 0, 0) and (1, 1, 1), respectively for both *likelihood* and *peril*.

Table 3. Complete data set for basic risk items for the evaluation of final aggregative risk

Basic risk items	Definition	$r_{i,j}^{k}$	$l_{i,j}^{k}$	$x = TFN_L$
$X_{1,1}^{4'}$	External source of contamination in storage tank	5	9	[0.21, 0.32, 0.45]
$X_{L_{2,1}}^{4'}$	Internal source of contamination in storage tank	1	3	[0, 0, 0.3]
$X_{3,2}^{4'}$	Contamination caused by broken pipes and gaskets	5	9	[0.21, 0.32, 0.45]
$X_{4,2}^{4'}$	Contamination during maintenance events	2	8	[0, 0.07, 0.16]
$X_{5,2}^{4'}$	Contamination caused by cross connection	6	7	[0.2, 0.32, 0.42]
$X_{3,2}^{3'}$	Regrowth of biofilm in tanks and resuspension	3	8	[0.06, 0.14, 0.24]
$X_{4,2}^{3'}$	Regrowth of biofilm in pipes and sloughing	2	7	[0, 0.06, 0.14]
$X_{6,5}^{4'}$	Disinfection byproducts coming through treated water	5	10	[0.24, 0.36, 0.5]
$X_{7,5}^{4'}$	Residual concentration of disinfectants	7	4	[0.1, 0.18, 0.28]
$X_{8,5}^{4'}$	Residues of other treatment chemicals	5	2	[0, 0.04, 0.1]
$X_{9,5}^{4'}$	Trace chemicals of source water	3	7	[0.05, 0.12, 0.21]
$X_{6,3}^{3'}$	Injured and escaped organisms in water treatment	2	10	[0, 0.09, 0.2]
$X_{7,4}^{3'}$	Elastomers	0	8	[0, 0, 0]
$X_{8,4}^{3'}$	Organic pollutants	0	5	[0, 0, 0]
$X_{10,9}^{4'}$	Leaching of pipe material	4	7	[0.1, 0.18, 0.28]
$X_{11,9}^{4'}$	Release of corrosion byproducts	8	9	[0.42, 0.56, 0.72]
$X_{10,5}^{3'}$	Leaching from liners and sealers in storage tank	3	5	[0.03, 0.08, 0.15]